AN5286

Precision Accelerometer Calibration

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Document information

Info	Content
Abstract	This application note documents the mathematics underlying the functions located in the Sensor Fusion Library and contained in the file *precisionAccelerometer.c*.

Revision history

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1. Introduction

1.1 Summary

This application note documents the mathematics for precision accelerometer calibration used by the NXP Sensor Fusion Library. The functions are listed in Table 1 and located in the sensor fusion file *precisionAccelerometer.c*. These functions perform the optional high precision calibration of an accelerometer sensor for gain, offset, cross-axis and rotation errors introduced during the mounting of the accelerometer on the circuit board.

The techniques described here achieve high precision without the need for expensive test equipment. The mathematics is closely related to that developed for the calibration of the magnetometer sensor in AN5019 "*Magnetic Calibration Algorithms*" and proofs are reused from that document where possible.

Sections 3 to 4 contain the mathematics for fitting three accelerometer calibration models of increasing complexity. Section 6 contains example error surface plots showing the results of applying the algorithms to the NXP FXOS8700 accelerometer. Section 7 documents the calculation of a rotation matrix to ensure zero tilt angle at the measurement taken when the product is level.

1.2 Terminology

Symbol	Definition
Left superscript G	Denotes that the measurement is in the global frame: ${}^{G}\boldsymbol{G}_k$
Left superscript S	Denotes that the measurement is in the sensor frame: ${}^{S}\boldsymbol{G}_k$
$\boldsymbol{A}$	Ellipsoid matrix: $\boldsymbol{A} = (\boldsymbol{W}^{-1})^T\boldsymbol{W}^{-1}$
g	Magnitude of the gravity vector ${}^{G}\boldsymbol{g}_0$ with magnitude 1 *g* by definition.
${}^{G}\boldsymbol{g}_0, {}^{S}\boldsymbol{g}_0$	Gravity vector in the global and sensor frames.
${}^{G}\boldsymbol{G}_k, {}^{S}\boldsymbol{G}_k$	Uncalibrated accelerometer measurement k in the global and sensor frames.
${}^{G}\boldsymbol{G}_{c,k}, {}^{S}\boldsymbol{G}_{c,k}$	Calibrated accelerometer measurement k in the global and sensor frames.
M	Number of measurements used in calibration fit
r_k	Error residual in measurement k
$\boldsymbol{R}$	Rotation matrix defining the product orientation.
$\Delta\boldsymbol{R}$	Rotation matrix defining the orientation of the accelerometer relative to the end product.
$\boldsymbol{V}$	Accelerometer calibration offset vector
$\boldsymbol{W}$	Accelerometer calibration gain matrix
$\boldsymbol{X}$	Matrix of accelerometer measurements
$\boldsymbol{\beta}$	Solution vector

1.3 Software Functions

Table 1. Sensor Fusion software functions

Functions	Description	Reference
`void fComputeAccelCalibration4(struct AccelBuffer *pthisAccelBuffer, struct AccelCalibration *pthisAccelCal, struct AccelSensor* pthisAccel)`	Determines the coefficients of the 4 parameter accelerometer calibration model.	4
`void fComputeAccelCalibration7(struct AccelBuffer *pthisAccelBuffer, struct AccelCalibration *pthisAccelCal, struct AccelSensor* pthisAccel)`	Determines the coefficients of the 7 parameter accelerometer calibration model.	5
`void fComputeAccelCalibration10(struct AccelBuffer *pthisAccelBuffer, struct AccelCalibration *pthisAccelCal, struct AccelSensor* pthisAccel)`	Determines the coefficients of the 10 parameter accelerometer calibration model.	6

2. Differences From Magnetic Calibration

The four-, seven- and ten-element accelerometer calibration models described in this document have two minor differences from the equivalent four-, seven- and ten-element magnetic calibration models described in AN5019 *"Magnetic Calibration Algorithms*".

The first difference is that the geomagnetic field strength B varies between 27 μT and 65 μT (more than 240 %) over the earth's surface whereas the gravitational field strength g varies by just 0.7 %. The magnetic calibration algorithms therefore solve for B on the assumption that the soft iron gain matrix has unit determinant implying no shielding or amplification of the geomagnetic field. The accelerometer calibration models, in contrast, fit the local gravitational field to have magnitude exactly 1 gravity by scaling the gain matrix determinant appropriately.

The second difference is that the geomagnetic inclination angle δ varies from –90º at the south geomagnetic pole to +90º at the north geomagnetic pole and is unknown without knowledge of one's location. The gravity vector in the global frame ${}^{G}\boldsymbol{g}_0$ always points exactly downwards at all locations and defines the local zero tilt plane. It is therefore straightforward in the accelerometer calibrations algorithms to define one measurement as representing *flat* and computing the rotation correction matrix that will force zero tilt angle irrespective of the mounting angle of the accelerometer in the final product. This is documented in Section 7.

The application of the algorithms also differs between the magnetic and accelerometer calibration cases. The magnetometer measurements are unaffected by acceleration and can therefore be taken while the product is moving and rotating. Typically several hundred magnetometer measurements are used to compute the magnetic calibration. The accelerometer, in contrast, is sensitive to acceleration as well as the applied gravitational field and so its calibration measurements must be taken when the accelerometer is stationary, isolated from vibration and the measurements then averaged to minimize measurement noise. Typically, 12 high-quality measurements are used to compute the precision accelerometer offset, gain and cross-axis calibration corrections.

Both the magnetic and accelerometer calibration algorithms have the tremendous advantage that they use sensor measurements taken at random, but different, orientations and do not require mechanical jigs or other techniques to force specific measurement orientations. A very high-quality accelerometer calibration can therefore be performed easily with no specialist equipment.

3. Four Parameter Accelerometer Calibration Model

3.1 Derivation of the Least Squares Solution

This section documents the accelerometer calibration algorithm implemented in function `fComputeAccelCalibration4` which calculates the four parameters comprising i) the accelerometer offset vector $\boldsymbol{V}$ and ii) a single gain correction factor W applied to all three accelerometer axes.

The actual accelerometer measurement ${}^{S}\boldsymbol{G}_k$ is modeled in terms of the true calibrated measurement ${}^{S}\boldsymbol{G}_{c,k}$ as:

$$ {}^{S}\boldsymbol{G}_k = \boldsymbol{W}\Delta\boldsymbol{R}\,{}^{S}\boldsymbol{G}_{c,k} + \boldsymbol{V} = W\Delta\boldsymbol{R}\,{}^{S}\boldsymbol{G}_{c,k} + \boldsymbol{V} \quad (1) $$

The term W models a single gain correction term common to all three axes and $\boldsymbol{V}$ models the accelerometer offsets in the three axes. For this model, the gain matrix $\boldsymbol{W} = W\boldsymbol{I}$:

$$ \boldsymbol{W} = W\boldsymbol{I} = \begin{pmatrix} W & 0 & 0 \\ 0 & W & 0 \\ 0 & 0 & W \end{pmatrix} \quad (2) $$

The left superscript S denotes that the measurements are defined in the sensor frame of reference.

The rotation matrix $\Delta\boldsymbol{R}$ models the tilt rotation error of the accelerometer inside the final product. The $\Delta\boldsymbol{R}$ notation implies that the tilt rotation error is small but this need not be the case and the tilt error could be 90º or 180º if the accelerometer circuit board is mounted at right angles or inverted in the final product.

If the calibration coefficients are known then the calibrated measurement ${}^{S}\boldsymbol{G}_{c,k}$ is obtained by inverting equation (1):

$$ {}^{S}\boldsymbol{G}_{c,k} = (\Delta\boldsymbol{R})^T W^{-1}\left({}^{S}\boldsymbol{G}_k - \boldsymbol{V}\right) \quad (3) $$

In the absence of acceleration, equation (1) for the three coordinate systems can be written as:

$$ {}^{S}\boldsymbol{G}_k = W\Delta\boldsymbol{R}\,{}^{S}\boldsymbol{g}_0 + \boldsymbol{V} = W\Delta\boldsymbol{R}\boldsymbol{R}\,{}^{G}\boldsymbol{g}_0 + \boldsymbol{V}\ (Aerospace, Windows\ 8) \quad (4) $$

$$ {}^{S}\boldsymbol{G}_k = -W\Delta\boldsymbol{R}\,{}^{S}\boldsymbol{g}_0 + \boldsymbol{V} = -W\Delta\boldsymbol{R}\boldsymbol{R}\,{}^{G}\boldsymbol{g}_0 + \boldsymbol{V}\ (Android) \quad (5) $$

The term ${}^{S}\boldsymbol{g}_0 = \boldsymbol{R}\,{}^{G}\boldsymbol{g}_0$ is the downwards pointing gravity vector in the global frame ${}^{G}\boldsymbol{g}_0$ rotated into the sensor frame by the product's orientation matrix $\boldsymbol{R}$. This term therefore represents the true applied gravitational stimulus on the accelerometer in the sensor frame assuming no physical acceleration.

The reason for the differing signs in equations (4) and (5) is that:

- the NED / Aerospace coordinate system is NED with the *z* axis pointing downwards whereas the Android and Windows 8 coordinate systems are ENU with the *z* axis pointing upwards
- the NED / Aerospace and the Windows 8 coordinate systems are gravity positive whereas the Android coordinate system is acceleration positive.

Both rotation matrices and the different sign conventions can be eliminated from equations (4) and (5) to give the accelerometer measurement locus for all three coordinate systems as:

$$W^{-2}\left({}^S\boldsymbol{G}_k - \boldsymbol{V}\right)^T\left({}^S\boldsymbol{G}_k - \boldsymbol{V}\right) = \left(\Delta\boldsymbol{R}\boldsymbol{R}\,{}^G\boldsymbol{g}_0\right)^T\left(\Delta\boldsymbol{R}\boldsymbol{R}\,{}^G\boldsymbol{g}_0\right) = \left({}^G\boldsymbol{g}_0\right)^T\boldsymbol{R}^T(\Delta\boldsymbol{R})^T\Delta\boldsymbol{R}\boldsymbol{R}\,{}^G\boldsymbol{g}_0 = g^2 \quad \textbf{(6)}$$

where the gravitational field strength g is defined as:

$$g = \left|{}^G\boldsymbol{g}_0\right| \quad \textbf{(7)}$$

If the accelerometer measurements are in units of gravity then $g = 1$.

Equation (6) models the locus of the accelerometer measurements ${}^S\boldsymbol{G}_k$ as lying on the surface of a sphere with radius Wg equal to W in units of ***g*** with its center offset from the origin by $\boldsymbol{V}$.

The mathematics which follows derives the offset vector $\boldsymbol{V}$ and gain term W using an approach almost identical to that used for the four element magnetic calibration. The calculation of the rotation correction matrix $\Delta\boldsymbol{R}$ is discussed in Section 7.

The error residual r_k for the *k-th* accelerometer measurement is defined in terms of the deviation of the calibrated measurement from the 1 ***g*** sphere as:

$$r_k = W^2\left(\left|{}^S\boldsymbol{G}_{c,k}\right|^2 - g^2\right) = \left\{(\Delta\boldsymbol{R})^T\left({}^S\boldsymbol{G}_k - \boldsymbol{V}\right)\right\}^T\left\{(\Delta\boldsymbol{R})^T\left({}^S\boldsymbol{G}_k - \boldsymbol{V}\right)\right\} - W^2g^2 \quad \textbf{(8)}$$

$$= {}^S\boldsymbol{G}_k{}^T\,{}^S\boldsymbol{G}_k - 2\,{}^S\boldsymbol{G}_k{}^T\boldsymbol{V} + \boldsymbol{V}^T\boldsymbol{V} - W^2g^2 \quad \textbf{(9)}$$

Equation (9) is identical to equation (64) in AN5019 “*Magnetic Calibration Algorithms*” with the term Wg replacing the geomagnetic field strength B. The least squares solution from AN5019 “*Magnetic Calibration Algorithms*” can therefore be re-used without additional proof.

The solution vector $\boldsymbol{\beta}$ from M accelerometer measurements is:

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 2V_x \\ 2V_y \\ 2V_z \\ W^2 g^2 - {V_x}^2 - {V_y}^2 - {V_z}^2 \end{pmatrix} = (\boldsymbol{X}^T \boldsymbol{X})^{-1} \boldsymbol{X}^T \boldsymbol{Y} \tag{10}$$

where:

$$\boldsymbol{X}^T \boldsymbol{X} = \sum_{k=0}^{M-1} \begin{pmatrix} {^S G_{x,k}}^2 & {^S G_{x,k}}\,{^S G_{y,k}} & {^S G_{x,k}}\,{^S G_{z,k}} & {^S G_{x,k}} \\ {^S G_{x,k}}\,{^S G_{y,k}} & {^S G_{y,k}}^2 & {^S G_{y,k}}\,{^S G_{z,k}} & {^S G_{y,k}} \\ {^S G_{x,k}}\,{^S G_{z,k}} & {^S G_{y,k}}\,{^S G_{z,k}} & {^S G_{z,k}}^2 & {^S G_{z,k}} \\ {^S G_{x,k}} & {^S G_{y,k}} & {^S G_{z,k}} & 1 \end{pmatrix} \tag{11}$$

$$\boldsymbol{X}^T \boldsymbol{Y} = \sum_{k=0}^{M-1} \begin{pmatrix} {^S G_{x,k}} \left({^S G_{x,k}}^2 + {^S G_{y,k}}^2 + {^S G_{z,k}}^2 \right) \\ {^S G_{y,k}} \left({^S G_{x,k}}^2 + {^S G_{y,k}}^2 + {^S G_{z,k}}^2 \right) \\ {^S G_{z,k}} \left({^S G_{x,k}}^2 + {^S G_{y,k}}^2 + {^S G_{z,k}}^2 \right) \\ \left({^S G_{x,k}}^2 + {^S G_{y,k}}^2 + {^S G_{z,k}}^2 \right) \end{pmatrix} \tag{12}$$

3.2 Offset Vector and Gain Matrix

The solution for the offset vector $\boldsymbol{V}$ and gain correction term W in terms of the solution vector $\boldsymbol{\beta}$ comes directly from equation (10) as:

$$\boldsymbol{V} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \left(\frac{1}{2}\right) \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} \tag{13}$$

$$Wg = \sqrt{\beta_3 + {V_x}^2 + {V_y}^2 + {V_z}^2} \tag{14}$$

The term g in equation (14) ensures dimensional consistency between the left and right hand sides since W is dimensionless. If the native units of gravities are used for the accelerometer measurements then $g = 1$.

4. Seven Parameter Magnetic Calibration Model

4.1 Derivation of the Least Squares Solution

This section documents the accelerometer calibration algorithm implemented in function `fComputeAccelCalibration7` which is closely related to the corresponding seven parameter magnetic calibration function.

The seven parameter magnetic calibration function performs an eigen-decomposition on a 7x7 measurement matrix to fit i) the three elements of the hard iron offset vector $\boldsymbol{V}$ ii) the three diagonal gain elements of the soft iron gain matrix $\boldsymbol{W}$ and iii) the geomagnetic field strength B. Strictly speaking, the seven parameter magnetic calibration has six degrees of freedom, not seven, since the gain matrix $\boldsymbol{W}$ is constrained to have unit determinant.

The seven parameter accelerometer calibration model uses essentially identical mathematics to fit i) the three elements of the accelerometer offset vector $\boldsymbol{V}$ and ii) the three diagonal elements of the accelerometer gain matrix $\boldsymbol{W}$ to iii) the local gravitational field strength g. The gain matrix $\boldsymbol{W}$ is not normalized to unit determinant because the gravitational field can be assumed to have magnitude of exactly 1 $\boldsymbol{g}$. The accelerometer calibration model, therefore, also has six degrees of freedom but is also termed a seven-parameter model to emphasize the closeness with the seven parameter magnetic calibration model.

The model for the actual sensor frame accelerometer measurement ${}^{S}\boldsymbol{G}_k$ in terms of the true calibrated measurement ${}^{S}\boldsymbol{G}_{c,k}$ is:

$$ {}^{S}\boldsymbol{G}_k = \boldsymbol{W}\Delta\boldsymbol{R}\,{}^{S}\boldsymbol{G}_{c,k} + \boldsymbol{V} \tag{15}$$

The gain matrix $\boldsymbol{W}$ is diagonal and can correct each of the three channel gains independently:

$$ \boldsymbol{W} = \begin{pmatrix} W_{xx} & 0 & 0 \\ 0 & W_{yy} & 0 \\ 0 & 0 & W_{zz} \end{pmatrix} \tag{16}$$

The calibrated measurement ${}^{S}\boldsymbol{G}_{c,k}$ is obtained by inverting equation (15):

$$ {}^{S}\boldsymbol{G}_{c,k} = (\Delta\boldsymbol{R})^T \boldsymbol{W}^{-1}\left({}^{S}\boldsymbol{G}_k - \boldsymbol{V}\right) \tag{17}$$

In the absence of acceleration, the applied stimulus is the gravity vector rotated into the sensor frame:

$$ {}^{S}\boldsymbol{G}_k = \boldsymbol{W}\Delta\boldsymbol{R}\,{}^{S}\boldsymbol{g}_0 + \boldsymbol{V} = \boldsymbol{W}\Delta\boldsymbol{R}\boldsymbol{R}\,{}^{G}\boldsymbol{g}_0 + \boldsymbol{V}\ (Aerospace, Windows\ 8) \tag{18}$$

$$ {}^{S}\boldsymbol{G}_k = -\boldsymbol{W}\Delta\boldsymbol{R}\,{}^{S}\boldsymbol{g}_0 + \boldsymbol{V} = -\boldsymbol{W}\Delta\boldsymbol{R}\boldsymbol{R}\,{}^{G}\boldsymbol{g}_0 + \boldsymbol{V}\ (Android) \tag{19}$$

The measurement locus for all three coordinate systems is:

$$ \left({}^{S}\boldsymbol{G}_k - \boldsymbol{V}\right)^T (\boldsymbol{W}^{-1})^T \boldsymbol{W}^{-1}\left({}^{S}\boldsymbol{G}_k - \boldsymbol{V}\right) = \left({}^{S}\boldsymbol{G}_k - \boldsymbol{V}\right)^T \boldsymbol{A}\left({}^{S}\boldsymbol{G}_k - \boldsymbol{V}\right) = g^2 \tag{20}$$

Equation (20) defines an ellipsoid aligned along the Cartesian axes with axes determined by the ellipsoid matrix $\boldsymbol{A}$ which is defined in the same manner as for magnetic calibration as:

$$\boldsymbol{A} = (\boldsymbol{W}^{-1})^T \boldsymbol{W}^{-1} \tag{21}$$

The error residual r_k for the k^{th} accelerometer measurement is defined in terms of the deviation of the calibrated measurement from the 1 g sphere as:

$$r_k = \left| {}^{S}\boldsymbol{G}_{c,k} \right|^2 - g^2 = \left\{ \boldsymbol{W}^{-1}\left({}^{S}\boldsymbol{G}_k - \boldsymbol{V} \right) \right\}^T \left\{ \boldsymbol{W}^{-1}\left({}^{S}\boldsymbol{G}_k - \boldsymbol{V} \right) \right\} - g^2 \tag{22}$$

$$= \left({}^{S}\boldsymbol{G}_k \right)^T \boldsymbol{A}\, {}^{S}\boldsymbol{G}_k - 2\left({}^{S}\boldsymbol{G}_k \right)^T \boldsymbol{A}\boldsymbol{V} + \boldsymbol{V}^T \boldsymbol{A}\boldsymbol{V} - g^2 \tag{23}$$

Equation (23) has identical form to equation (91) in AN5019 "*Magnetic Calibration Algorithms*" with g replacing B allowing the least squares solution vector $\boldsymbol{\beta}$ to be defined as:

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{pmatrix} = \begin{pmatrix} A_{xx} \\ A_{yy} \\ A_{zz} \\ -2A_{xx}V_x \\ -2A_{yy}V_y \\ -2A_{zz}V_z \\ A_{xx}{V_x}^2 + A_{yy}{V_y}^2 + A_{zz}{V_z}^2 - g^2 \end{pmatrix} \tag{24}$$

and determined as the eigenvector with smallest eigenvalue of the measurement matrix $\boldsymbol{X}^T\boldsymbol{X}$ defined for M measurements as:

$$\boldsymbol{X}^T\boldsymbol{X} = \sum_{k=0}^{M-1} \begin{pmatrix}
{}^{S}G_{x,k}{}^4 & {}^{S}G_{x,k}{}^2\,{}^{S}G_{y,k}{}^2 & {}^{S}G_{x,k}{}^2\,{}^{S}G_{z,k}{}^2 & {}^{S}G_{x,k}{}^3 & {}^{S}G_{x,k}{}^2\,{}^{S}G_{y,k} & {}^{S}G_{x,k}{}^2\,{}^{S}G_{z,k} & {}^{S}G_{x,k}{}^2 \\
{}^{S}G_{y,k}{}^2\,{}^{S}G_{x,k}{}^2 & {}^{S}G_{y,k}{}^4 & {}^{S}G_{y,k}{}^2\,{}^{S}G_{z,k}{}^2 & {}^{S}G_{y,k}{}^2\,{}^{S}G_{x,k} & {}^{S}G_{y,k}{}^3 & {}^{S}G_{y,k}{}^2\,{}^{S}G_{z,k} & {}^{S}G_{y,k}{}^2 \\
{}^{S}G_{z,k}{}^2\,{}^{S}G_{x,k}{}^2 & {}^{S}G_{z,k}{}^2\,{}^{S}G_{y,k}{}^2 & {}^{S}G_{z,k}{}^4 & {}^{S}G_{z,k}{}^2\,{}^{S}G_{x,k} & {}^{S}G_{z,k}{}^2\,{}^{S}G_{y,k} & {}^{S}G_{z,k}{}^3 & {}^{S}G_{z,k}{}^2 \\
{}^{S}G_{x,k}{}^3 & {}^{S}G_{x,k}\,{}^{S}G_{y,k}{}^2 & {}^{S}G_{x,k}\,{}^{S}G_{z,k}{}^2 & {}^{S}G_{x,k}{}^2 & {}^{S}G_{x,k}\,{}^{S}G_{y,k} & {}^{S}G_{x,k}\,{}^{S}G_{z,k} & {}^{S}G_{x,k} \\
{}^{S}G_{y,k}\,{}^{S}G_{x,k}{}^2 & {}^{S}G_{y,k}{}^3 & {}^{S}G_{y,k}\,{}^{S}G_{z,k}{}^2 & {}^{S}G_{y,k}\,{}^{S}G_{x,k} & {}^{S}G_{y,k}{}^2 & {}^{S}G_{y,k}\,{}^{S}G_{z,k} & {}^{S}G_{y,k} \\
{}^{S}G_{z,k}\,{}^{S}G_{x,k}{}^2 & {}^{S}G_{z,k}\,{}^{S}G_{y,k}{}^2 & {}^{S}G_{z,k}{}^3 & {}^{S}G_{z,k}\,{}^{S}G_{x,k} & {}^{S}G_{z,k}\,{}^{S}G_{y,k} & {}^{S}G_{z,k}{}^2 & {}^{S}G_{z,k} \\
{}^{S}G_{x,k}{}^2 & {}^{S}G_{y,k}{}^2 & {}^{S}G_{z,k}{}^2 & {}^{S}G_{x,k} & {}^{S}G_{y,k} & {}^{S}G_{z,k} & 1
\end{pmatrix} \tag{25}$$

4.2 Offset Vector and Gain Matrix

The ellipsoid fit matrix $\boldsymbol{A}$ is obtained directly from the first three rows of the solution vector $\boldsymbol{\beta}$ in equation (24):

$$\boldsymbol{A} = \begin{pmatrix} A_{xx} & 0 & 0 \\ 0 & A_{yy} & 0 \\ 0 & 0 & A_{zz} \end{pmatrix} = \begin{pmatrix} \beta_0 & 0 & 0 \\ 0 & \beta_1 & 0 \\ 0 & 0 & \beta_2 \end{pmatrix} \tag{26}$$

The eigenvector returned by the mathematical software is normalized but undetermined in sign and can be negated without invalidating the solution. If β_0, β_1 and β_2 are negative then the entire solution vector $\boldsymbol{\beta}$ is first negated giving the equally valid eigenvector with the same minimum eigenvalue but pointing in the opposite direction.

The solution for the inverse gain matrix $\boldsymbol{W}^{-1}$ in terms of the ellipsoid matrix $\boldsymbol{A}$ is given by equation (21):

$$\boldsymbol{W}^{-1} = \sqrt{\boldsymbol{A}} = \sqrt{\begin{pmatrix} A_{xx} & 0 & 0 \\ 0 & A_{yy} & 0 \\ 0 & 0 & A_{zz} \end{pmatrix}} = \begin{pmatrix} \sqrt{\beta_0} & 0 & 0 \\ 0 & \sqrt{\beta_1} & 0 \\ 0 & 0 & \sqrt{\beta_2} \end{pmatrix} \quad \textbf{(27)}$$

The offset vector $\boldsymbol{V}$ is given by equation (24) as:

$$\begin{pmatrix} -2A_{xx}V_x \\ -2A_{yy}V_y \\ -2A_{zz}V_z \end{pmatrix} = \begin{pmatrix} \beta_3 \\ \beta_4 \\ \beta_5 \end{pmatrix} \Rightarrow \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \begin{pmatrix} \left(\frac{-\beta_3}{2A_{xx}}\right) \\ \left(\frac{-\beta_4}{2A_{yy}}\right) \\ \left(\frac{-\beta_5}{2A_{zz}}\right) \end{pmatrix} = \begin{pmatrix} \left(\frac{-\beta_3}{2\beta_0}\right) \\ \left(\frac{-\beta_4}{2\beta_1}\right) \\ \left(\frac{-\beta_5}{2\beta_2}\right) \end{pmatrix} \quad \textbf{(28)}$$

The offset vector $\boldsymbol{V}$ is unaffected by the negation of the solution vector $\boldsymbol{\beta}$ since both numerator and denominator terms in equation (28) are negated with cancelling effect.

The last row of equation (24) gives the gravitational field strength consistent g with the computed gain matrix $\boldsymbol{W}$:

$$g = \sqrt{A_{xx}{V_x}^2 + A_{yy}{V_y}^2 + A_{zz}{V_z}^2 - \beta_6} \quad \textbf{(29)}$$

At this point the solution is self-consistent between the gain matrix $\boldsymbol{W}$ and the gravitational field strength g in the sense that the solution could state that the approximately 1 *g* accelerometer measurements result from being on the moon with one sixth of a gravity accompanied by an accelerometer gain error in $\boldsymbol{W}$ of 6x.

The final step in the seven-parameter accelerometer calibration algorithm therefore scales the gain matrix $\boldsymbol{W}$ to give $g = 1$ exactly (assuming the measurements are in units of gravities) as:

$$\boldsymbol{W}^{-1} \leftarrow \left(\frac{1}{g}\right)\boldsymbol{W}^{-1} = \left(\frac{1}{\sqrt{A_{xx}{V_x}^2 + A_{yy}{V_y}^2 + A_{zz}{V_z}^2 - \beta_6}}\right)\boldsymbol{W}^{-1} \quad \textbf{(30)}$$

This is slightly different from the final step in the magnetic calibration algorithms which set the ellipsoid matrix $\boldsymbol{A}$ to have unit determinant and scales the fitted geomagnetic field strength B by the equivalent amount.

5. Ten Parameter Accelerometer Calibration Model

5.1 Derivation of the Least Squares Solution

This section documents the accelerometer calibration algorithm implemented in function `fComputeAccelCalibration10` which is closely related to the corresponding ten-parameter magnetic calibration function.

The ten-parameter magnetic calibration function performs an eigen-decomposition on a 10x10 measurement matrix to fit i) the three elements of the hard iron offset vector $\boldsymbol{V}$ ii) the three diagonal gain elements of the soft iron gain matrix $\boldsymbol{W}$ iii) the three symmetric off-diagonal cross-axis elements of $\boldsymbol{W}$ and iv) the geomagnetic field strength B. Strictly speaking, the ten-parameter magnetic calibration has nine degrees of freedom, not ten, since the gain matrix $\boldsymbol{W}$ is constrained to have unit determinant.

The ten-parameter accelerometer calibration model fits i) the three elements of the accelerometer offset vector $\boldsymbol{V}$ ii) the three diagonal gain elements of the gain matrix $\boldsymbol{W}$ iii) the three off-diagonal cross-axis terms in $\boldsymbol{W}$ and iv) the local gravitational field strength g. The gain matrix $\boldsymbol{W}$ is not normalized to unit determinant because the gravitational field if fitted to have magnitude of exactly 1 *g*. The accelerometer calibration model therefore also has nine degrees of freedom but is also termed a *ten-parameter model* to emphasize the closeness with the ten-parameter magnetic calibration model.

The model for the actual sensor frame accelerometer measurement ${}^{S}\boldsymbol{G}_k$ in terms of the true sensor frame calibrated measurement ${}^{S}\boldsymbol{G}_{c,k}$ is:

$$ {}^{S}\boldsymbol{G}_k = \boldsymbol{W}\Delta\boldsymbol{R}\,{}^{S}\boldsymbol{G}_{c,k} + \boldsymbol{V} \tag{31} $$

The gain matrix $\boldsymbol{W}$ is symmetric in the 10 parameter model with six independent coefficients:

$$ \boldsymbol{W} = \begin{pmatrix} W_{xx} & W_{xy} & W_{xz} \\ W_{xy} & W_{yy} & W_{yz} \\ W_{xz} & W_{yz} & W_{zz} \end{pmatrix} \tag{32} $$

The calibrated measurement ${}^{S}\boldsymbol{G}_{c,k}$ is obtained by inverting equation (31):

$$ {}^{S}\boldsymbol{G}_{c,k} = (\Delta\boldsymbol{R})^T\boldsymbol{W}^{-1}\left({}^{S}\boldsymbol{G}_k - \boldsymbol{V}\right) \tag{33} $$

In the absence of acceleration, the applied stimulus is again the gravity vector rotated into the sensor frame:

$$ {}^{S}\boldsymbol{G}_k = \boldsymbol{W}\Delta\boldsymbol{R}\,{}^{S}\boldsymbol{g}_0 + \boldsymbol{V} = \boldsymbol{W}\Delta\boldsymbol{R}\boldsymbol{R}\,{}^{G}\boldsymbol{g}_0 + \boldsymbol{V}\ (Aerospace, Windows\ 8) \tag{34} $$

$$ {}^{S}\boldsymbol{G}_k = -\boldsymbol{W}\Delta\boldsymbol{R}\,{}^{S}\boldsymbol{g}_0 + \boldsymbol{V} = -\boldsymbol{W}\Delta\boldsymbol{R}\boldsymbol{R}\,{}^{G}\boldsymbol{g}_0 + \boldsymbol{V}\ (Android) \tag{35} $$

The measurement locus for all three coordinate systems is again:

$$\left({}^{S}\boldsymbol{G}_k - \boldsymbol{V}\right)^T (\boldsymbol{W}^{-1})^T \boldsymbol{W}^{-1}\left({}^{S}\boldsymbol{G}_k - \boldsymbol{V}\right) = \left({}^{S}\boldsymbol{G}_k - \boldsymbol{V}\right)^T \boldsymbol{A}\left({}^{S}\boldsymbol{G}_k - \boldsymbol{V}\right) = g^2 \tag{36}$$

where:

$$\boldsymbol{A} = (\boldsymbol{W}^{-1})^T \boldsymbol{W}^{-1} \tag{37}$$

Since $\boldsymbol{W}$ has off-diagonal elements, equation (36) defines the surface of an ellipsoid with arbitrary orientation relative to the Cartesian axes.

The error residual r_k for the k^{th} accelerometer measurement is again defined in terms of the deviation of the calibrated measurement from the 1*g* sphere as:

$$r_k = \left|{}^{S}\boldsymbol{G}_{c,k}\right|^2 - g^2 = \left\{\boldsymbol{W}^{-1}\left({}^{S}\boldsymbol{G}_k - \boldsymbol{V}\right)\right\}^T \left\{\boldsymbol{W}^{-1}\left({}^{S}\boldsymbol{G}_k - \boldsymbol{V}\right)\right\} - g^2 \tag{38}$$

$$= \left({}^{S}\boldsymbol{G}_k\right)^T \boldsymbol{A}\, {}^{S}\boldsymbol{G}_k - 2\left({}^{S}\boldsymbol{G}_k\right)^T \boldsymbol{A}\boldsymbol{V} + \boldsymbol{V}^T\boldsymbol{A}\boldsymbol{V} - g^2 \tag{39}$$

Equation (39) has identical form to equation (111) in AN5019 “*Magnetic Calibration Algorithms*” with g replacing B allowing the least squares solution vector $\boldsymbol{\beta}$ to be defined as:

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_7 \\ \beta_8 \\ \beta_9 \end{pmatrix} = \begin{pmatrix} A_{xx} \\ A_{xy} \\ A_{xz} \\ A_{yy} \\ A_{yz} \\ A_{zz} \\ -2A_{xx}V_x - 2A_{xy}V_y - 2A_{xz}V_z \\ -2A_{xy}V_x - 2A_{yy}V_y - 2A_{yz}V_z \\ -2A_{xz}V_x - 2A_{yz}V_y - 2A_{zz}V_z \\ A_{xx}{V_x}^2 + 2A_{xy}V_xV_y + 2A_{xz}V_xV_z + A_{yy}{V_y}^2 + 2A_{yz}V_yV_z + A_{zz}{V_z}^2 - g^2 \end{pmatrix} \tag{40}$$

and determined as the eigenvector with smallest eigenvalue of the measurement matrix $\boldsymbol{X}^T\boldsymbol{X}$ defined for M measurements as:

$$\boldsymbol{X}^T\boldsymbol{X} = \sum_{k=0}^{M-1}\begin{pmatrix} {}^{S}G_{x,k}{}^{4} & 2\,{}^{S}G_{x,k}{}^{3}\,{}^{S}G_{y,k} & 2\,{}^{S}G_{x,k}{}^{3}\,{}^{S}G_{z,k} & \dots & {}^{S}G_{x,k}{}^{2}\,{}^{S}G_{z,k} & {}^{S}G_{x,k}{}^{2} \\ 2\,{}^{S}G_{x,k}{}^{3}\,{}^{S}G_{y,k} & 4\,{}^{S}G_{x,k}{}^{2}\,{}^{S}G_{y,k}{}^{2} & 4\,{}^{S}G_{x,k}{}^{2}\,{}^{S}G_{y,k}\,{}^{S}G_{z,k} & \dots & 2\,{}^{S}G_{x,k}\,{}^{S}G_{y,k}\,{}^{S}G_{z,k} & 2\,{}^{S}G_{x,k}\,{}^{S}G_{y,k} \\ 2\,{}^{S}G_{x,k}{}^{3}\,{}^{S}G_{z,k} & 4\,{}^{S}G_{x,k}{}^{2}\,{}^{S}G_{y,k}\,{}^{S}G_{z,k} & 4\,{}^{S}G_{x,k}{}^{2}\,{}^{S}G_{z,k}{}^{2} & \dots & 2\,{}^{S}G_{x,k}\,{}^{S}G_{z,k}{}^{2} & 2\,{}^{S}G_{x,k}\,{}^{S}G_{z,k} \\ {}^{S}G_{x,k}{}^{2}\,{}^{S}G_{y,k}{}^{2} & 2\,{}^{S}G_{x,k}\,{}^{S}G_{y,k}{}^{3} & 2\,{}^{S}G_{x,k}\,{}^{S}G_{y,k}{}^{2}\,{}^{S}G_{z,k} & \dots & {}^{S}G_{y,k}{}^{2}\,{}^{S}G_{z,k} & {}^{S}G_{y,k}{}^{2} \\ 2\,{}^{S}G_{x,k}{}^{2}\,{}^{S}G_{y,k}\,{}^{S}G_{z,k} & 4\,{}^{S}G_{x,k}\,{}^{S}G_{y,k}{}^{2}\,{}^{S}G_{z,k} & 4\,{}^{S}G_{x,k}\,{}^{S}G_{y,k}\,{}^{S}G_{z,k}{}^{2} & \dots & 2\,{}^{S}G_{y,k}\,{}^{S}G_{z,k}{}^{2} & 2\,{}^{S}G_{y,k}\,{}^{S}G_{z,k} \\ {}^{S}G_{x,k}{}^{2}\,{}^{S}G_{z,k}{}^{2} & 2\,{}^{S}G_{x,k}\,{}^{S}G_{y,k}\,{}^{S}G_{z,k}{}^{2} & 2\,{}^{S}G_{x,k}\,{}^{S}G_{z,k}{}^{3} & \dots & {}^{S}G_{z,k}{}^{3} & {}^{S}G_{z,k}{}^{2} \\ {}^{S}G_{x,k}{}^{3} & 2\,{}^{S}G_{x,k}{}^{2}\,{}^{S}G_{y,k} & 2\,{}^{S}G_{x,k}{}^{2}\,{}^{S}G_{z,k} & \dots & {}^{S}G_{x,k}\,{}^{S}G_{z,k} & {}^{S}G_{x,k} \\ {}^{S}G_{x,k}{}^{2}\,{}^{S}G_{y,k} & 2\,{}^{S}G_{x,k}\,{}^{S}G_{y,k}{}^{2} & 2\,{}^{S}G_{x,k}\,{}^{S}G_{y,k}\,{}^{S}G_{z,k} & \dots & {}^{S}G_{y,k}\,{}^{S}G_{z,k} & {}^{S}G_{y,k} \\ {}^{S}G_{x,k}{}^{2}\,{}^{S}G_{z,k} & 2\,{}^{S}G_{x,k}\,{}^{S}G_{y,k}\,{}^{S}G_{z,k} & 2\,{}^{S}G_{x,k}\,{}^{S}G_{z,k}{}^{2} & \dots & {}^{S}G_{z,k}{}^{3} & {}^{S}G_{z,k} \\ {}^{S}G_{x,k}{}^{2} & 2\,{}^{S}G_{x,k}\,{}^{S}G_{y,k} & 2\,{}^{S}G_{x,k}\,{}^{S}G_{z,k} & \dots & {}^{S}G_{z,k} & 1 \end{pmatrix} \tag{41}$$

5.2 Offset Vector and Gain Matrix

The ellipsoid fit matrix $\boldsymbol{A}$ is computed from the first six rows of the solution vector $\boldsymbol{\beta}$ in equation (40):

$$\boldsymbol{A} = \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{xy} & A_{yy} & A_{yz} \\ A_{xz} & A_{yz} & A_{zz} \end{pmatrix} = \begin{pmatrix} \beta_0 & \beta_1 & \beta_2 \\ \beta_1 & \beta_3 & \beta_4 \\ \beta_2 & \beta_4 & \beta_5 \end{pmatrix} \tag{42}$$

The solution eigenvector $\boldsymbol{\beta}$ is undefined within a multiplicative factor of ±1 (assuming it is normalized to unit magnitude). Since physically sensible solutions for $\boldsymbol{A}$ require that it have a positive determinant, the entire solution vector $\boldsymbol{\beta}$ is negated if $|\boldsymbol{A}| < 0$.

The inverse gain matrix $\boldsymbol{W}^{-1}$ is computed from the square root of the symmetric matrix $\boldsymbol{A}$ as described in equations (132) to (134) in AN5019 "*Magnetic Calibration Algorithms*".

$$\boldsymbol{W}^{-1} = \sqrt{\boldsymbol{A}} = \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{xy} & A_{yy} & A_{yz} \\ A_{xz} & A_{yz} & A_{zz} \end{pmatrix}^{\frac{1}{2}} = \begin{pmatrix} \beta_0 & \beta_1 & \beta_2 \\ \beta_1 & \beta_3 & \beta_4 \\ \beta_2 & \beta_4 & \beta_5 \end{pmatrix}^{\frac{1}{2}} \tag{43}$$

The offset vector $\boldsymbol{V}$ is given by:

$$\boldsymbol{V} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = -\left(\frac{1}{2}\right)\begin{pmatrix} \beta_0 & \beta_1 & \beta_2 \\ \beta_1 & \beta_3 & \beta_4 \\ \beta_2 & \beta_4 & \beta_5 \end{pmatrix}^{-1}\begin{pmatrix} \beta_6 \\ \beta_7 \\ \beta_8 \end{pmatrix} = -\left(\frac{1}{2}\right)\boldsymbol{A}^{-1}\begin{pmatrix} \beta_6 \\ \beta_7 \\ \beta_8 \end{pmatrix} \tag{44}$$

The last row of equation (40) gives the gravitational field strength consistent g with the computed gain matrix $\boldsymbol{W}$:

$$g = \sqrt{A_{xx}V_x{}^2 + 2A_{xy}V_xV_y + 2A_{xz}V_xV_z + A_{yy}V_y{}^2 + 2A_{yz}V_yV_z + A_{zz}V_z{}^2 - \beta_9} \tag{45}$$

The final step in the ten-parameter accelerometer calibration algorithm therefore scales the gain matrix $\boldsymbol{W}$ to give $g = 1$ exactly:

$$\boldsymbol{W}^{-1} \leftarrow \left(\frac{1}{g}\right)\boldsymbol{W}^{-1} = \left(\frac{1}{\sqrt{A_{xx}{V_x}^2 + 2A_{xy}V_xV_y + 2A_{xz}V_xV_z + A_{yy}{V_y}^2 + 2A_{yz}V_yV_z + A_{zz}{V_z}^2 - \beta_9}}\right)\boldsymbol{W}^{-1} \tag{46}$$

6. Example Calibration Surfaces

This section contains example plots showing the radial error from the 1 *g* sphere $\left|{}^{S}\boldsymbol{G}_{c,k}\right| - g$ measured using the accelerometer on an NXP FXOS8700 before and after re-calibration. The accelerometer was mounted on an industrial robot and measurements taken at 3º intervals in roll angle between –180º and 180º and pitch angles between –90º and 90º. This angle range covers the entire 1 *g* sphere. The error surface is defined on the 1 *g* sphere but is plotted on a flat cylindrical map projection for simplicity.

Fig 1 measured on an FXOS8700 shows an error surface typical for a consumer accelerometer. The error surface is dominated by offset and gain errors and has range –50 mg to +50 mg. Fine structure from the signal processing chain is just visible in the measurements.

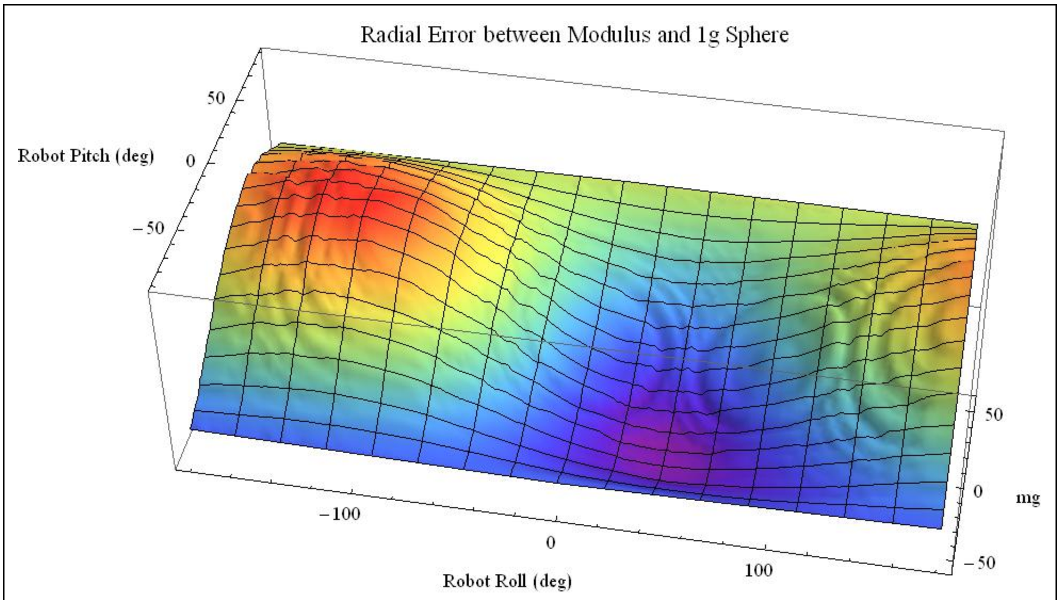

Fig 1. FXOS8700 error surface before re-calibration

Fig 2 shows the same FXOS8700 measurements after re-calibration using the four-parameter model. The offset errors are eliminated but the assumption of a common gain correction term W for all three axes is not terribly accurate. The error residual has, however, reduced from 50 mg to 10 mg.

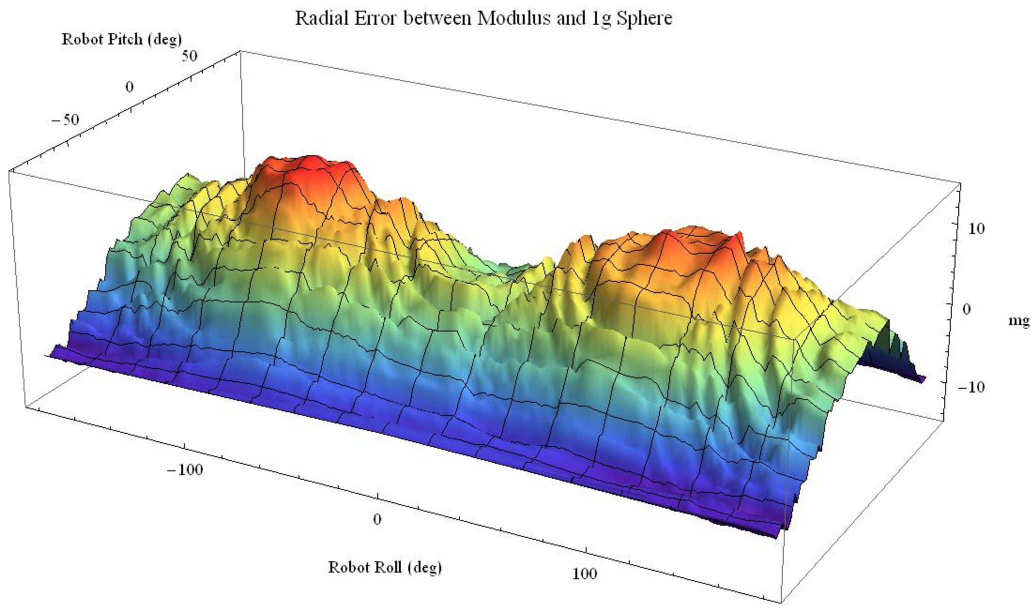

Fig 2. FXOS8700 error surface after four-parameter re-calibration

Fig 3 shows the same FXOS8700 measurements after re-calibration using the seven-parameter model. The gain errors in the three accelerometer channels are now independently corrected reducing the radial error to just 5 mg. The remaining red hot spots result from cross-axis errors which are not corrected in the seven-parameter model.

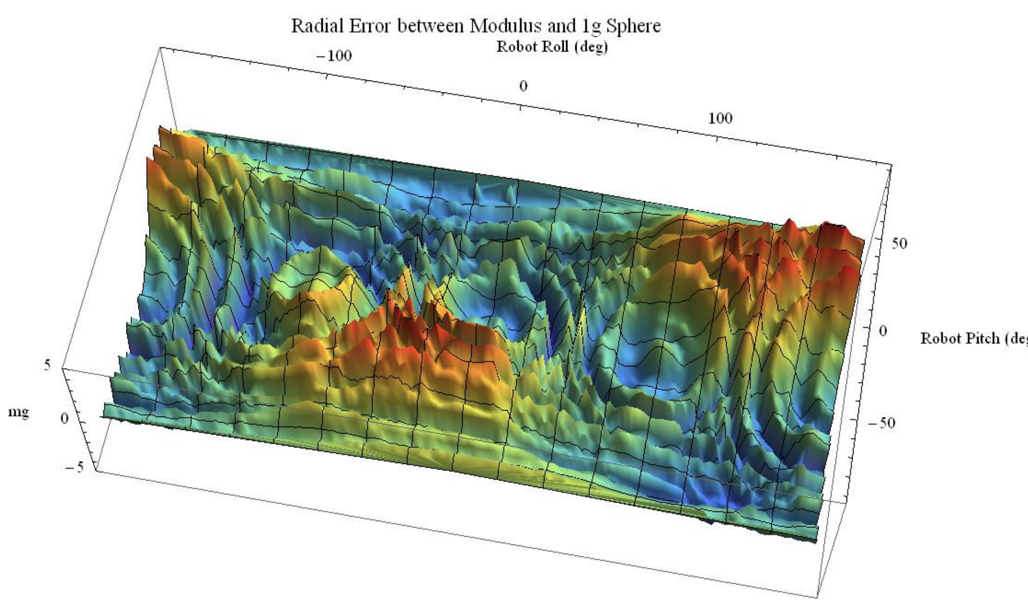

Fig 3. FXOS8700 error surface after seven-parameter re-calibration

Fig 4 shows the same FXOS8700 measurements after re-calibration using the ten-parameter model. The cross-axis errors present after calibration with the seven-parameter model are eliminated and the error residual is reduced to 2.5 mg. The remaining structure results from non-linearities in the MEMS structure response and the accelerometer ADC signal processing chain.

There are three points on the 1 *g* sphere where the three x, y and z channels have value +1 *g* and three points where the three channels have value –1 *g*. The error surface therefore approximates six-fold symmetry which appears as four symmetrical points along the zero pitch "equator" and two additional symmetrical points at the 90º pitch "north pole" and –90º pitch "south pole". The north and south pole structure is, however, "opened out" and distorted by the cylindrical projection.

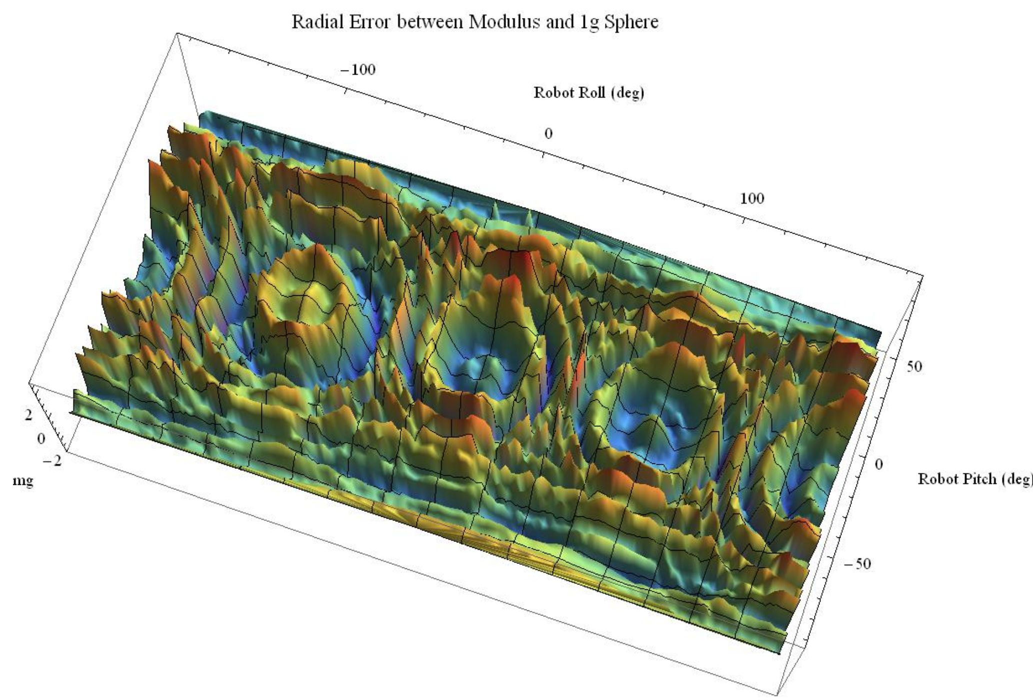

Fig 4. FXOS8700 error surface after ten-parameter re-calibration

7. Calculation of the Tilt Correction Matrix

The tilt rotation matrix $\Delta\boldsymbol{R}$ is very easily determined by ensuring that one measurement is taken when the final product is flat and then using the mathematics in AN5021 "*Calculation of Orientation Matrices from Sensor Data*" to compute the tilt matrix for that measurement.

Section 4 of AN5021 "*Calculation of Orientation Matrices from Sensor Data"* gives the NED, Android and Windows 8 tilt matrices, now computed from the calibrated measurement ${}^{S}\boldsymbol{G}_{c,0}$ at the flat measurement labeled zero, as:

$$\Delta \boldsymbol{R}_{NED} = \frac{1}{\left| {}^{S}\boldsymbol{G}_{c,0} \right|} \begin{pmatrix} \sqrt{{}^{S}G_{cy,0}{}^{2} + {}^{S}G_{cz,0}{}^{2}} & 0 & {}^{S}G_{cx,0} \\ \dfrac{-\,{}^{S}G_{cx,0}\,{}^{S}G_{cy,0}}{\sqrt{{}^{S}G_{cy,0}{}^{2} + {}^{S}G_{cz,0}{}^{2}}} & \dfrac{{}^{S}G_{cz,0}\left| {}^{S}\boldsymbol{G}_{c,0} \right|}{\sqrt{{}^{S}G_{cy,0}{}^{2} + {}^{S}G_{cz,0}{}^{2}}} & {}^{S}G_{cy,0} \\ \dfrac{-\,{}^{S}G_{cx,0}\,{}^{S}G_{cz,0}}{\sqrt{{}^{S}G_{cy,0}{}^{2} + {}^{S}G_{cz,0}{}^{2}}} & \dfrac{-\,{}^{S}G_{cy,0}\left| {}^{S}\boldsymbol{G}_{c,0} \right|}{\sqrt{{}^{S}G_{cy,0}{}^{2} + {}^{S}G_{cz,0}{}^{2}}} & {}^{S}G_{cz,0} \end{pmatrix} \tag{47}$$

$$\Delta \boldsymbol{R}_{Android} = \frac{1}{\left| {}^{S}\boldsymbol{G}_{c,0} \right|} \begin{pmatrix} \sqrt{{}^{S}G_{cy,0}{}^{2} + {}^{S}G_{cz,0}{}^{2}} & 0 & {}^{S}G_{cx,0} \\ \dfrac{-\,{}^{S}G_{cx,0}\,{}^{S}G_{cy,0}}{\sqrt{{}^{S}G_{cy,0}{}^{2} + {}^{S}G_{cz,0}{}^{2}}} & \dfrac{{}^{S}G_{cz,0}\left| {}^{S}\boldsymbol{G}_{c,0} \right|}{\sqrt{{}^{S}G_{cy,0}{}^{2} + {}^{S}G_{cz,0}{}^{2}}} & {}^{S}G_{cy,0} \\ \dfrac{-\,{}^{S}G_{cx,0}\,{}^{S}G_{cz,0}}{\sqrt{{}^{S}G_{cy,0}{}^{2} + {}^{S}G_{cz,0}{}^{2}}} & \dfrac{-\,{}^{S}G_{cy,0}\left| {}^{S}\boldsymbol{G}_{c,0} \right|}{\sqrt{{}^{S}G_{cy,0}{}^{2} + {}^{S}G_{cz,0}{}^{2}}} & {}^{S}G_{cz,0} \end{pmatrix} \tag{48}$$

$$\Delta \boldsymbol{R}_{Win8} = \frac{1}{\left| {}^{S}\boldsymbol{G}_{c,0} \right|} \begin{pmatrix} \dfrac{\left| {}^{S}\boldsymbol{G}_{c,0} \right|}{\sqrt{1 + \left(\dfrac{{}^{S}G_{cx,0}}{{}^{S}G_{cz,0}} \right)^{2}}} & \dfrac{{}^{S}G_{cx,0}\,{}^{S}G_{cy,0}}{{}^{S}G_{cz,0}\sqrt{1 + \left(\dfrac{{}^{S}G_{cx,0}}{{}^{S}G_{cz,0}} \right)^{2}}} & -\,{}^{S}G_{cx,0} \\ 0 & -\,{}^{S}G_{cz,0}\sqrt{1 + \left(\dfrac{{}^{S}G_{cx,0}}{{}^{S}G_{cz,0}} \right)^{2}} & -\,{}^{S}G_{cy,0} \\ \dfrac{-\,{}^{S}G_{cx,0}\left| {}^{S}\boldsymbol{G}_{c,0} \right|}{{}^{S}G_{cz,0}\sqrt{1 + \left(\dfrac{{}^{S}G_{cx,0}}{{}^{S}G_{cz,0}} \right)^{2}}} & \dfrac{{}^{S}G_{cy,0}}{\sqrt{1 + \left(\dfrac{{}^{S}G_{cx,0}}{{}^{S}G_{cz,0}} \right)^{2}}} & -\,{}^{S}G_{cz,0} \end{pmatrix} \tag{49}$$

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9. List of tables

10. Contents

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